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Denying The Liar *

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Abstract. The liar paradox is standardly supposed to arise from three conditions: classical bivalent truth value semantics, the Tarskian truth schema, and the formal constructability of a sentence that says of itself that it is not true. Standard solutions to the paradox, beginning most notably again with Tarski, try to forestall the paradox by rejecting or weakening one or more of these three conditions. It is argued that all efforts to avoid the liar paradox by watering down any of the three assumptions suffers serious disadvantages that are at least as undesirable as the liar paradox itself. Instead, a new solution is proposed that admits that if the liar sentence is true then it is false, in the first paradox dilemma horn, but denies that the liar sentence is true, but asserting instead that it is false, and refuting the second paradox dilemma horn according to which it is supposed to follow that if the liar sentence is false then it is true. The reasoning for the second paradox dilemma horn is flawed, in that it is not only not supported by but actually contradicted by the Tarskian truth schema. We could only infer the second dilemma horn if it were to classically follow from the assumption that the liar sentence is false, and from the three liar paradox conditions, that therefore it is false that the liar sentence is false. This entire sentence can be shown to be false on the basis of the standard truth schema, thus blocking the paradox. Alternative formulations of the liar sentence are discussed, and the formal proofs and counterproofs for the two paradox dilemma horns, are considered along with the further philosophical implications of maintaining a resolute stance that the liar sentence is simply false.

The liar paradox is said to arise as a consequence of accepting three conditions that taken individually seem theoretically unproblematic. The assumptions include:

(1) The adequacy of the bivalent truth value semantics of classical logic to represent the internal logical structure of truth value predications. We can formalize commitment to bivalent logic by the principle:

(CL) $\forall p[\mathrm{TRUE}\ulcorner p\urcorner \leftrightarrow \neg\mathrm{FALSE}\ulcorner p\urcorner]$.[1]

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[1] Angle brackets in the metalinguistic semantic sentences, 'TRUE$\ulcorner L\urcorner$' and 'FALSE$\ulcorner L\urcorner$,' superficially resemble Quinean corner quotations, which are

(2) The standard Tarskian truth schema for interpreting the truth values of sentences according to a broadly interpreted correspondence, disquotational or deflationary concept of truth, whereby:

(TS) $\forall p[\mathrm{TRUE}\ulcorner p \urcorner \leftrightarrow p]$.

(3) The constructibility of the liar sentence, which says of itself that it is false; declaring, in effect, 'I am FALSE,' 'This sentence is FALSE,' or L: FALSE$\ulcorner L \urcorner$.

There are several ways of formalizing the liar sentence; if necessary, where type theoretical restrictions apply, it can be done by Gödel-arithmetizing the syntax of a sentence attributing falsehood to a Gödel coded sentence N that is itself coded as Gödel number N. For simplicity, we symbolize the liar sentence as the conditional:

(L) $L \rightarrow \mathrm{FALSE}\ulcorner L \urcorner$.[2]

The informal derivation of the liar paradox proceeds by dilemma from the classical tautology, L or not-L. If L, then, according to the standard truth schema, L is true; but then L is false, since L says that L is false; hence, by the truth schema, not-L. If not-L, then, again, according to the truth schema, L is false; from which, since L says that L is false, it is supposed to be false that L is false; this is to say that L is true, from which it is supposed to follow from the truth schema that not-L. Thus, liar sentence L in a classical logical framework is true if and only if it is false, L iff not-L.

Solutions to the liar generally focus on one or more of the three conditions and relax or modify it in such a way that the paradox is forestalled. Without surveying the proposals for avoiding the paradox in these categories, I want to argue that despite the persuasive appeal of the informal characterization, the paradox does not formally obtain even where all three conditions are accepted. The paradox goes through in the first true-to-false dilemma horn, but not in the second false-to-true horn. I identify precisely where the informal exposition of the second dilemma horn plays fast and loose with the needed inference that $\mathrm{FALSE}\ulcorner L \urcorner \rightarrow \mathrm{TRUE}\ulcorner L \urcorner$; in effect, the inference whereby

conventionally used to indicate an intensional context where intersubstitution of coreferential terms or logically equivalent sentences fails *salva veritate*. The brackets here in contrast with some applications do not convert a syntax item into its name, but serve only to set off sentences to which a truth value is extensionally attributed.

[2] Readers expecting a biconditional instead of conditional formulation of the liar sentence should turn immediately to the section, *Addendum on the Biconditional Liar*.

$\text{FALSE}\ulcorner L \urcorner \rightarrow \text{FALSE}\ulcorner\text{FALSE}\ulcorner L \urcorner\urcorner$, from which we could otherwise classically derive the conclusion $\text{TRUE}\ulcorner L \urcorner$. Although the inference appears valid when it is informally explained, if we slow down the action and look more closely at the logic required to sustain the second liar paradox dilemma horn, we discover that its reasoning is fatally flawed, that it is unsupported and actually contradicted by the standard truth schema.

First, consider the unimpeded dilemma horn that begins with the assumption L in the tautology $L \vee \neg L$ and proceeds to the conclusion $\neg L$. The inference takes this form, if we spell out every step explicitly to indicate its reliance on the three liar paradox conditions, (CL), (TS) and (L):

First Liar Paradox Dilemma horn:

1.	L	Assumption
2.	$L \rightarrow \text{FALSE}\ulcorner L \urcorner$	(L) Definition of L
3.	$\text{FALSE}\ulcorner L \urcorner$	1,2 CL
4.	$\forall p[\text{TRUE}\ulcorner p \urcorner \leftrightarrow p]$	(TS) Truth Schema
5.	$\text{TRUE}\ulcorner L \urcorner \leftrightarrow L$	4 Universal Instantiation
6.	$\forall p[\text{TRUE}\ulcorner p \urcorner \leftrightarrow \neg\text{FALSE}\ulcorner p \urcorner]$	(CL) Classical Logic
7.	$\text{TRUE}\ulcorner L \urcorner \leftrightarrow \neg\text{FALSE}\ulcorner L \urcorner$	6 Universal Instantiation
8.	$\neg\text{TRUE}\ulcorner L \urcorner$	3,7 CL
9.	$\neg L$	5,8 CL
10.	$L \rightarrow \neg L$	1-9 CL

Thus, the first paradox dilemma horn goes through unhindered. Things are different when we turn to the second paradox dilemma horn based on the assumption of the second disjunct $\neg L$ in the tautology $L \vee \neg L$. The argument begins in this way, slowed down again so that every step can be considered in detail:

Second (Incomplete) Liar Paradox Dilemma horn:

1.	$\neg L$	Assumption
2.	$\forall p[\text{TRUE}\ulcorner p \urcorner \leftrightarrow p]$	(TS) Truth Schema
3.	$\text{TRUE}\ulcorner L \urcorner \leftrightarrow L$	2 Universal Instantiation
4.	$\neg\text{TRUE}\ulcorner L \urcorner$	1,3 CL
5.	$\forall p[\text{TRUE}\ulcorner p \urcorner \leftrightarrow \neg\text{FALSE}\ulcorner p \urcorner]$	(CL) Classical Logic
6.	$\text{TRUE}\ulcorner L \urcorner \leftrightarrow \neg\text{FALSE}\ulcorner L \urcorner$	5 Universal Instantiation
7.	$\text{FALSE}\ulcorner L \urcorner$	4,6 CL

So far, so good. In the informal exposition, if we freeze things at this point, this is precisely the juncture at which the inference trades on the fact that the meaning or internal content of liar sentence L declares or materially implies that L is false. If it is false that the liar sentence is false, then, classically, the liar sentence is true; hence, if the liar sentence is false then it is true, by which the standard truth schema appears to support the conclusion L; so that now, apparently, we should also get the second dilemma horn, $\neg L \rightarrow L$.

The trouble is that the inference is formally blocked after step (7), given the standard Tarskian truth schema, so that the second half of the paradox dilemma cannot be formally validly deduced. From step (7), if we were formally to track the informal explanation of the second horn, we would need to expand the sentence $\mathrm{FALSE}\ulcorner L \urcorner$ to $\mathrm{FALSE}\ulcorner \mathrm{FALSE}\ulcorner L \urcorner\urcorner$, on the strength of the fact that liar sentence L says of itself that it is false. From (6) and (7) we can classically infer $\neg\mathrm{TRUE}\ulcorner L \urcorner$. This does not yet provide the necessary iterated falsehood attributions to L, whereby the truth of L follows from its falsehood, and L follows from $\neg L$. The inference from $\mathrm{FALSE}\ulcorner L \urcorner$ to $\mathrm{FALSE}\ulcorner \mathrm{FALSE}\ulcorner L \urcorner\urcorner$ and thence to $\mathrm{TRUE}\ulcorner L \urcorner$ requires a freewheeling substitution of sentence $\mathrm{FALSE}\ulcorner L \urcorner$ for sentence L in $\mathrm{FALSE}\ulcorner L \urcorner$ itself, on the grounds that L *means* $\mathrm{FALSE}\ulcorner L \urcorner$. In classical logic, however, we could *only* authorize the expansion of $\mathrm{FALSE}\ulcorner L \urcorner$ to $\mathrm{FALSE}\ulcorner \mathrm{FALSE}\ulcorner L \urcorner\urcorner$ by substitution involving the definition of the liar sentence if it were true that $\mathrm{FALSE}\ulcorner L \urcorner \rightarrow \mathrm{FALSE}\ulcorner \mathrm{FALSE}\ulcorner L \urcorner\urcorner$. Far from this conditional holding true in the framework of three conditions we have considered, the proposition is false and its negation is formally derivable from the standard truth schema. The anti-theorem is proved by the following inference:

Antitheorem to Block Second Liar Paradox Dilemma horn:

1.	$\mathrm{FALSE}\ulcorner L \urcorner$	(7) Above
2.	$\forall p[\mathrm{TRUE}\ulcorner p \urcorner \leftrightarrow p]$	(TS) Truth Schema
3.	$\mathrm{TRUE}\ulcorner L \urcorner \leftrightarrow L$	2 Universal Instantiation
4.	$\mathrm{TRUE}\ulcorner \mathrm{FALSE}\ulcorner L \urcorner\urcorner$	1,3 CL
5.	$\forall p[\mathrm{TRUE}\ulcorner p \urcorner \leftrightarrow \neg\mathrm{FALSE}\ulcorner p \urcorner]$	(CL) Classical Logic
6.	$\mathrm{TRUE}\ulcorner L \urcorner \leftrightarrow \neg\mathrm{FALSE}\ulcorner L \urcorner$	5 Universal Instantiation
7.	$\neg\mathrm{FALSE}\ulcorner \mathrm{FALSE}\ulcorner L \urcorner\urcorner$	4,6 CL
8.	$\mathrm{FALSE}\ulcorner L \urcorner \wedge \neg\mathrm{FALSE}\ulcorner \mathrm{FALSE}\ulcorner L \urcorner\urcorner$	1,7 CL
9.	$\neg[\mathrm{FALSE}\ulcorner L \urcorner \rightarrow \mathrm{FALSE}\ulcorner \mathrm{FALSE}\ulcorner L \urcorner\urcorner]$	8 CL

The liar paradox is thwarted by the failure of the second $\neg L \rightarrow L$ dilemma horn. We cannot validly deduce L from $\neg L$, because we cannot validly deduce $\mathrm{TRUE}\ulcorner L \urcorner$ from $\mathrm{FALSE}\ulcorner L \urcorner$. For, as we have now proved, we cannot validly deduce $\mathrm{FALSE}\ulcorner \mathrm{FALSE}\ulcorner L \urcorner\urcorner$ from $\mathrm{FALSE}\ulcorner L \urcorner$. For the same reason, substituting FALSE$\ulcorner L \urcorner$ for L in FALSE$\ulcorner L \urcorner$ to obtain FALSE$\ulcorner$FALSE$\ulcorner L \urcorner\urcorner$ (classically equivalent to TRUE$\ulcorner L \urcorner$) is also deductively invalid. The substitution's invalidity leaves us with the conclusion that FALSE$\ulcorner L \urcorner$, and blocks the inference of FALSE$\ulcorner$FALSE$\ulcorner L \urcorner\urcorner$ from FALSE$\ulcorner L \urcorner$. The liar paradox is forestalled if we deny the liar, holding that the liar sentence is simply false, full stop.

And why should we not? The liar sentence *says* that it is false. Why not take it at its word? If, at some level, the liar sentence intuitively or conceptually if not formally logically or deductively entails a contradiction or inconsistency, that is only another reason to evaluate the liar sentence as false, to deny its truth. Informal exposition of the liar paradox, particularly in its second false-to-true horn, deceptively makes it appear a matter of course to deduce that the liar sentence is true from the assumption that it is false. We have now seen formally that this is not the case, but that the principle needed in order to uphold the inference is not only unavailable to justify the second dilemma horn, but its negation is forthcoming directly from the standard truth schema together with the assumption that the liar sentence is false.

The conclusion of the first dilemma horn, that if the liar sentence is true then it is false, is also readily explained by denying the liar. If the liar sentence is evaluated as false without further semantic oscillation from false to true and true to false, then from the definition of the material conditional it follows trivially in classical logic that if the liar sentence is true, then it is also false. If, formally, it cannot further be validly deduced that if the liar sentence is false then it is true, then there is no liar paradox. The liar paradox as we have defined the liar sentence does not contradict classical logic, the standard truth schema, or raise concerns about the formal constructibility of the liar sentence.[3]

[3] It should be needless to say but is still worth mentioning that there are and can be no genuine logical paradoxes. Paradoxes occur only in our thinking and sometimes in our clumsy language. If there were genuine paradoxes in the sense of outright logical antinomies, then, contrary to fact, the actual world we manifestly inhabit would not be a logically possible world; it would be logically impossible by virtue of containing a logical inconsistency. That, *a fortiori*, cannot happen; what are called logical and semantic paradoxes are always solvable. Sometimes paradoxes can be blunted by logical analysis of the language in which they are formulated, sometimes by enforcing previously unappreciated distinctions and reforming language or revising principles that appear at least superficially to be conceptually unproblematic until their consequences are investigated. What are called logical paradoxes provoke us to unravel our reasoning, clarify our ideas, look closely at the formal inferences that are supposed to lead to inconsistency, and find exactly where things have gone wrong. No matter how the analysis of a logical puzzle turns out, we enhance our understanding of the concepts involved in the paradox that we can make it our philosophical task to unconfuse.

Addendum on the Biconditional Liar

As we have seen, the second paradox dilemma horn appears at first to go through by substitution. We are told that informally L is just the sentence FALSE$\ulcorner L \urcorner$. Why not then substitute FALSE$\ulcorner L \urcorner$ for L in the second dilemma horn beginning with the sentence FALSE$\ulcorner L \urcorner$, to obtain FALSE$\ulcorner$FALSE$\ulcorner L \urcorner\urcorner$?

Truth value ascriptions are extensional contexts *par excellence*. The intersubstitution of logically equivalent sentences *salva veritate* should therefore be freely admitted to the purely extensional context, FALSE$\ulcorner\urcorner$. If making the substitution contradicts the conclusion of the antitheorem that is supposed to block the second dilemma horn, then so much the worse for it. If we are dealing with a genuine logical-semantic paradox, then contradictions might be rife throughout the inference chain. The substitution required for the second dilemma horn is nevertheless unavailable in the exposition we have considered, where the meaning relation by which liar sentence L is defined does not make L logically biconditionally equivalent to FALSE$\ulcorner L \urcorner$. For it is only on the strength of a biconditional equivalence that intersubstitutions *salva veritate* are authorized even in purely extensional contexts.

The formal dissolution of the liar paradox within classical logic that has been proposed provides for two different formalizations of the liar sentence. One statement is simply, L: FALSE$\ulcorner L \urcorner$, and the other has the conditional form, L: $L \rightarrow$ FALSE$\ulcorner L \urcorner$. If, however, the liar sentence is defined biconditionally as L^*: $L^* \leftrightarrow$ FALSE$\ulcorner L^* \urcorner$, then an outright logical antinomy is unavoidable. The inference then takes this course, availing ourselves *a fortiori* of a biconditional (L^*) version of the previously demonstrated first dilemma horn:

Biconditional Liar Paradox

1.	L^*	Assumption
2.	$\neg L^* \leftrightarrow \mathrm{FALSE}\ulcorner L^* \urcorner$	(L^*) Biconditional Liar
3.	$\neg\mathrm{FALSE}\ulcorner L^* \urcorner$	1,2 CL
4.	$\forall p[\mathrm{TRUE}\ulcorner p \urcorner \leftrightarrow \neg\mathrm{FALSE}\ulcorner p \urcorner]$	(CL) Classical Logic
5.	$\mathrm{TRUE}\ulcorner L^* \urcorner \leftrightarrow \neg\mathrm{FALSE}\ulcorner L^* \urcorner$	4 Universal Instantiation
6.	$\mathrm{TRUE}\ulcorner L^* \urcorner$	3,5 CL
7.	$\forall p[\mathrm{TRUE}\ulcorner p \urcorner \leftrightarrow p]$	(TS) Truth Schema
8.	$\mathrm{TRUE}\ulcorner L^* \urcorner \leftrightarrow L^*$	7 Universal Instantiation
9.	L^*	6,8 CL
10.	$\neg L^* \rightarrow L^*$	1-9 CL
11.	$L^* \leftrightarrow \neg L^*$	10 + First (L^*) Dilemma horn

The liar sentence cannot be defined biconditionally if the purpose is to produce an interesting derivation of the liar paradox. If we introduce the liar as (L^*), then, in light of the standard truth schema, we in effect assume that $\mathrm{TRUE}\ulcorner L^*\urcorner \leftrightarrow \mathrm{FALSE}\ulcorner L^*\urcorner$. The biconditional proposition, and from it the derivation of $L^* \leftrightarrow \neg L^*$, is no more paradoxical or in need of solution, given its blatantly logically antinomical form, than if we were to try to embarrass classical logic by baldly asserting any other explicit contradiction, such as $A \leftrightarrow \neg A$, of which the biconditional liar is merely a substitution instance.

We no more challenge the syntactical or semantic integrity of classical logic by introducing the biconditional liar sentence in (L^*) than we would by the deadpan assumption that $A \leftrightarrow \neg A$. The difference is reflected in the fact that the liar paradox is informally presented in terms of the sentence, 'This sentence is false,' from which a contradiction is supposed to follow—not, significantly, by means of any such blatantly overtly self-contradictory sentence as, 'This sentence is true and this sentence is false,' or 'This sentence is true if and only if it is false.' The problem posed by the liar paradox is whether a contradiction can be derived from the nonexplicitly contradictory sentence, 'This sentence is false,' given the standard truth schema in a classical bivalent semantics. This is undoubtedly why commentators on the liar have generally construed the liar sentence itself as a conditional rather than biconditional statement, among whose implications, if the paradox is genuine, would include the biconditional metalinguistic semantic assertion that the liar sentence is true if and only if it is false. A typical example is Brian Skyrms in his essay, 'Notes on Quantification and Self-Reference'(1970, especially pp. 70–4).[4]

We should, accordingly, consistently with the rationale for denying the liar, and as a requirement of truth evaluation in a classical logical framework, declare without further ado and without risking semantic instability that the biconditional liar sentence (L^*) is unequivocally false, on the grounds that $\mathrm{TRUE}\ulcorner L^*\urcorner \leftrightarrow \mathrm{FALSE}\ulcorner L^*\urcorner$ is an explicit logical inconsistency that by classical truth table analysis is unqualifiedly false. Avoiding triviality in the liar paradox by sticking to a conditional rather than biconditional formulation of the liar sentence at the same time precludes the logical equivalence needed for the valid substitution of $\mathrm{FALSE}\ulcorner L\urcorner$ for L in the second paradox dilemma horn.

[4] In diagnosing the liar, Skyrms focuses on the role of quotation in identity formulations of the liar ($a = \sim Ta$), and on the failure of substitution of referentially equivalent meanings of sentence a in intensional meaning contexts, resulting in meaningless sentences in a supervaluation semantics. His solution is altogether different than the one proposed here, where the relevant truth value attribution contexts are purely extensional.

References

Skyrms, B. (1970). Notes on Quantification and Self-Reference. In R.L. Martin (Ed.), *The Paradox of the Liar*. New Haven and London: Yale University Press.