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Liar Paradox and Substitution into Intensional Contexts

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Abstract. John Barker, in two recent essays, raises a variety of intriguing criticisms to challenge my interpretation of the liar paradox and the type of solution I propose in 'Denying the Liar' and 'Denying the Liar Reaffirmed.' Barker continues to believe that I have misunderstood the logical structure of the liar sentence and its expression, and that as a result my solution misfires. I shall try to show that on the contrary my analysis is correct, and that Barker does not properly grasp what my solution to the liar paradox involves. Additionally, I argue that Barker makes fundamental errors in the explanation of liar sentence formulations in intensional contexts and in the classical metatheory he invokes to support his criticisms.

1. Liar Paradox Redux

One cannot exhaust the logical and philosophical interest in the liar paradox, even when to some it seems through a variety of methods to have been finally resolved. In 'Disquotation, Conditionals, and the Liar' (Barker, 2009), John Barker takes a second stab at criticizing my solution to the liar in my essay, 'Denying the Liar' (Jacquette, 2007), which he first attacks in, 'Undeniably Paradoxical: Reply to Jacquette' (Barker, 2008), and again in my reply, 'Denying the Liar Reaffirmed' (Jacquette, 2008). Barker in his more recent essay has tightened up his arguments considerably, but I find the new formulations in which he purports to pay careful attention to the intensional contexts in which the liar sentence is expressed, together with his appeal to Gödel's incompleteness results, to be logically inconclusive when they are not simply mistaken.

Barker avoids reacting to the individual countercriticisms I offered in defense of my original solution to the liar paradox in (Jacquette, 2008), when he writes that: 'Jacquette has offered a veritable laundry-list of complaints about my original paper. It would be easy enough to respond point-by-point, but such a response would try my reader's patience' (Barker, 2009). My intention, I should mention, was not to take Barker to the cleaners, but rather to answer systematically the separate components of the criticism he made in his (2008), and I wish he had done the same here in response. Would it really try the patience of readers already interested in the intricacies of the liar paradox for Barker to have met my objections head-on, as he maintains, or does he find some of the objections I made rather difficult to answer, and accordingly chooses to move the discussion to different ground, strategically selecting only those arguments he thinks he can address?

2. Barker's Latest Criticisms

What Barker offers now as an objection of my solution to the liar has three primary components. He argues that: (1) In my treatment of the liar I have confused an important distinction, between the liar sentence (L) and a sentence that says of (L) that it is a liar sentence. (2) There are formulations of the liar sentence that by means of an innocent-seeming principle of disquotation produce logical contradictions that are not solved by my proposal for forestalling the liar. (3) Classical metatheory, especially a famous result of Gödel's, proves that Tarski's truth-schemata alone are responsible for deriving logical inconsistencies in Peano arithmetic, and that it is Tarski's truth-schemata that must therefore be held to blame in producing the as-yet unresolved liar.

Of these parts of Barker's new discussion, (1) and (2) introduce new and interesting, but I think inconclusive considerations, while (3) recapitulates an argument Barker made in his (2008). The latter of which I continue to think represents a serious misunderstanding of the first limiting metatheorem in Gödel's (1931). Unlike Barker, I shall try to meet his most recent criticisms point-for-point. I do not see how otherwise the dispute can fruitfully proceed or be meaningfully resolved. Briefly, my response to component (1) in Barker's (2009) is that his distinction between liar sentences and sentences that say that a sentence is the liar sentence is obviously correct, but that he misapplies it in criticizing my (2007) and (2008). As to what I am calling Barker's component (2), I think it is easy to show that the constructions he offers violate basic principles of valid intersubstitution of coreferential terms and logically equivalent sentences within intensional contexts *salva veritate*, despite his efforts to be sensitive to the intensionality of some syntactically complex formulas. Consequently, that no valid deductions of a liar paradox from a liar sentence are forthcoming by his method that are left unresolved by my solution. With reference to component (3), I think that Barker makes some rather basic mistakes in interpreting Gödel's 1931 proof, and that he misstates Gödel's results as demonstrating an outright logical inconsistency in Peano arithmetic. If I am right, then Barker's assertions in this regard are far too strong and unconditional when seen from a proper perspective as an important chapter in the history of mathematical logic.

3. What is a Liar Sentence? What Does it Try to Say?

The first problem to address is the correct logical formalization of a basic liar sentence. Barker in this part of his argument correctly draws a series of deductively valid conclusions from false and imprecise assumptions that are central to his criticisms of my treatment of the liar. Barker writes:

> As a first step toward formalizing Liar reasoning, we must find a way of saying, formally, that a given sentence is in fact a Liar sentence. At issue here are two candidate formalizations of this claim, both of which Jacquette considers in his original article. First, there is a conditional:
>
> (2) $L \rightarrow \neg\mathrm{TRUE}(\ulcorner L \urcorner)$
>
> (2) says, or attempts to say, that L is a Liar sentence. That is, (2) represents an attempt to express formally the following idea: that L says that L is not true. (Barker, 2009, p. 6)

It is crucial to see that from the outset Barker misrepresents my statement of the liar in both of my essays to which he refers. A comparison shows a number of discrepancies; some trivial, I think, but others absolutely crucial to Barker's interpretation of what I am saying, on which his criticism is based.

Note first that the numbering of what Barker ascribes to me as a statement of the liar sentence, or statement that a given statement is a liar sentence, is Barker's own. I did not write: (2) $L \rightarrow \neg\mathrm{TRUE}(\ulcorner L \urcorner)$; but rather: (L) $L \rightarrow \mathrm{FALSE}\ulcorner L \urcorner$. Ignoring the mysterious appearance of the parentheses around the intensional corner-quote specification of sentence L that seem to do no work at all in Barker's version, the crucial difference is that in my formulation sentence-designator (L) is not just the number of a proposition in a sequence of propositions I discuss, like Barker's (2), but rather the *entire* statement *defines* the liar sentence (L) as $L \rightarrow \mathrm{FALSE}\ulcorner L \urcorner$. Sentence (L) is thereby made the sentence that says *of itself* that sentence L materially implies that L is false. Conventionally, we should expect that (L) $\neq L$, as indicated by the difference in font type. The liar sentence (L) on this formulation is whatever sentence L materially implies its own falsehood, which interprets my understanding of what it means for a sentence to say of itself that it is false.

Thus, Barker gets it completely wrong when he next remarks: 'It should be noted that (2) is not itself a Liar sentence; rather, it is L that is the Liar sentence, and (2) simply says (or attempts to say) that L is a Liar sentence' (Barker, 2009, p. 6). A moment's reflection should show that Barker's distinction here cannot possibly be correct, because the sentence L by itself, a mere propositional symbol, as opposed to (L), does not have any internal logical structure. Sentence L as opposed to (L), defined as the entirety of

$L \rightarrow \text{FALSE}\ulcorner L \urcorner$, cannot by itself therefore imply or otherwise assert its own falsehood, and it cannot by itself therefore be a liar sentence. Contrary to what Barker asserts, it is (L) in its entirety that formulates the liar, at least as I introduce it in the exposition with which Barker proposes to take issue, presenting a sentence L that declares its own falsehood. The liar sentence as I understand it is most definitely not, as Barker would have it, sentence L as a subformula contained *within* (L), where L by itself serves only as a sentence variable. Thus, (L) defines the liar sentence in effect as *any* sentence that materially implies its own falsehood. Barker's (2) or my (L) do not say that L is a liar sentence; instead, (L) as I intend it is in fact supposed to explicate the logical form of an archetypal simplified liar sentence.

Barker compares his numbered conditional sentence (2) with a similar biconditional sentence (3) (a version of my original (L*)), which he presents as: (3) $L \leftrightarrow \neg\text{TRUE}(\ulcorner L \urcorner)$ (for the record, I offer: L*: $L^* \leftrightarrow \text{FALSE}\ulcorner L^* \urcorner$ (Jacquette, 2007, p. 96; and identically in Jacquette, 2008, p. 144)). The question in my original essay was whether the liar sentence could be formulated as a conditional in (L) or as a biconditional in (L*). I had shown that as a conditional the liar paradox based on (L) goes through only in one of its horns, and hence does not really embody a semantic inconsistency reflected in a syntactic contradiction. I argued consequently that the biconditional liar sentence (L*) could at best provide only a trivial, uninteresting basis for the liar paradox, because (L*) is itself internally syntactically self-contradictory. The point was that to posit (L*) without further ado as a biconditional statement of the liar sentence and then to derive the liar paradox from it with the usual classical apparatus was no more significant than formulating any other syntactical contradiction, $p \wedge \neg p$, declaring a paradox, and calling it a day. Barker next makes the same point with respect to his (3) as he does for (2):

> [T]here is an important clarification that needs to be made. The sentences (2) and (3) above are not Liar sentences. The sentence (2) represents an attempt to say that L is a Liar sentence; but (2) itself is not a Liar sentence. Likewise, sentence (3) represents an attempt to say that L is a Liar sentence; but (3) itself is not a Liar sentence. This ought to be fairly clear and straightforward; indeed, in my original paper I considered it too obvious to be worth mentioning. Yet Jacquette is not entirely consistent on this point. He spends a good deal of time in both papers, for example, speaking of "conditional liars" and "biconditional liars." Both terms are misnomers, however, since it is the sentences (2) and (3) that are conditionals and biconditionals, respectively, whereas it is the sentence L itself that is a Liar sentence. For all we know, the sentence L might itself be a conditional, biconditional, conjunction, atomic sentence, or anything else. (Barker, 2009, p. 7)

Again, I would never imagine that Barker's (2) or (3) or my definitions (L) or (L*) should be construed as saying that sentences L or L^* all by themselves are liar sentences, since I regard any such statement as false. L

and *L** are considered only as understructured to say of themselves that they are false, conditionally or biconditionally, or, for that matter, to say anything else about themselves.[1]

Barker, building on false assumptions, proceeds to misconstrue my *L* and *L** as having a hidden internal logical structure, that in the second component of his three-part strategy in (Barker, 2009) he proposes to unveil. As he puts it by way of preparation for such an excursus immediately above: 'For all we know, the sentence *L* might itself be a conditional, biconditional, conjunction, atomic sentence, or anything else.' This is not at all how I have presented *L* or *L**. My effort throughout has been to argue that (L) defines the liar sentence as occurring when and only when any sentence *L*, itself possessing no hidden or internal logical structure, materially implies or says of itself that it is false. The same is trivially and to that extent unsuccessfully attempted by the definition of the biconditional liar in (L*).

Barker says that in his previous critique he took it for granted that *L* and *L** rather than (L) and (L*) or his (2) or (3) were themselves liar sentences, with a hidden internal structure by virtue of which they somehow say of themselves that they are false. However, why would I leave the logical mechanism by which the liar sentence is supposed to produce the liar paradox hidden and unexplicated, when my purpose is to expose what I maintain is the inconclusive logical structure of the liar by which no genuine paradox is entailed? I can inform the interested reader that I never intended any such thing, but that my intention was rather to *display* in the logical structure of (L) the naked logical form of the liar sentence. This unequivocally makes (L) in its entirety rather than sentence *L*, the latter functioning only as a sentence variable, the definition of the liar sentence that I consider. Insofar as Barker misunderstands this and thinks that *L* by itself is the liar sentence, rather than the liar sentence being conditionally defined in the entirety of (L) (or of (L*) in the biconditional variation), he is barking up the wrong tree.

I was surprised to read Barker then continuing: 'Jacquette is not entirely consistent on this point. He spends a good deal of time in both papers, for example, speaking of "conditional liars" and "biconditional liars".' This is disconcerting because, of course, I am 100% consistent in speaking only of

[1] Barker (2009, pp. 7-8): 'The dispute between (2) [(L)] and (3) [(L*)] is not a dispute over whether Liar sentences are conditionals, biconditionals, or anything else. It is a dispute about whether the statement *that* a given sentence is a Liar is best seen as a conditional or biconditional.' This is very different from the way Barker speaks of the liar in his (2008, especially pp. 138-139) prior to my (2008) countercriticisms of his first wave of objections, in which he does indeed take the dispute to be whether liar sentences are conditionals or biconditionals and tries to make a case for the latter.

the conditional and biconditional liar sentences, as defined respectively by my (L) and (L*). Indeed, I never waver in any instance from my loyalty to these descriptions, and I certainly never wander into Barker's way of putting things by regarding L or L^*, the mere sentence variables, as themselves liar sentences with an opaque internal logical structure, the details about which we are left to speculate. Whether I am finally correct to do so or not, I am at least not inconsistent on the matter in either of the essays to which Barker refers. Rather, I consistently say precisely what Barker does not want me to say, what he thinks it is incorrect to say, but which I continue to regard as the only intelligible formulation of the liar.

I have already explained why I maintain that L and L^* considered in and of themselves, independently of satisfying the logical requirements of the definition of the liar sentence in (L), cannot themselves be formulations of the liar sentence, and why they need to be understood as nothing more than sentence variables in the liar sentence definition (L), where it is deductively inadequate to entail the liar paradox, and in (L*), where, as a statement leading to syntactical paradox, it is logically trivial. Once more with feeling: sentence L by itself can in one sense be construed as expressing the liar sentence, but only insofar as it satisfies all the conditions for L explicitly laid out in definition (L). Where Barker will eventually define (L*) as $L^* \leftrightarrow \neg\mathrm{TRUE}(\ulcorner L^* \urcorner)$, which he accepts as a statement of the liar sentence, I define the liar sentence (L) as $L \rightarrow \mathrm{FALSE}\ulcorner L \urcorner$. That is the liar sentence as I interpret it, in the logical form of a material conditional. There are historical, intuitive conceptual, and philosophical reasons for adopting the conditional reading of the liar sentence. To repeat the most important philosophical reason from my laundry list that Barker saves the reader tedium by refraining from answering, there is this. If we interpret the liar sentence in Barker's way, then the liar paradox amounts to nothing more than inferring one contradiction from another, which is not what logicians would ordinarily call paradoxical. The conditional interpretation of the logical structure of the liar sentence in definition (L), on the other hand, says of itself that it is false, and the logical structure whereby it purports to do so is fully exposed in (L) = $L \rightarrow \mathrm{FALSE}\ulcorner L \urcorner$.

Sentence (L) is not merely a sentence *about* the liar sentence. It *is* the self-referring liar sentence itself represented and defined by (L), a sentence that says of itself that it is false. The sentence does not say and is not intelligibly interpreted as saying of itself that it is true if and only if it is false, which would amount to the palpable assertion of a logical contradiction. We already know that if we have propositional negation among the logical operators of a language, then we have the possibility of formulating a logical contradiction. If that is all the liar paradox boils down to, then it loses all pretense of significance. The interesting question about the liar sentence has always been whether the mere fact that a liar sentence can be formulated in a language made strong enough to formulate a sentence that minimally says of itself that it is false can sustain a logical diagonalization with unwanted semantic consequences.[2] I assume that there can simply be no liar *paradox*, in the correct sense of the word, if the liar *sentence* from which the paradox is supposed to follow is itself an explicit logical contradiction like (L*). There is no paradox if a contradiction follows from another baldly posited contradiction, purely on the grounds that such a contradiction as (L*) is a wff of the language. The only alternative I see, after rejecting Barker's (2009) proposal, is my conditional formulation (L) by which I originally interpreted the liar (Jacquette, 2007), where (L) unlike (L*) is manifestly not strong enough classically to deduce the liar paradox. If the question of the liar paradox is whether a sentence's self-referentially denying its own truth is sufficient to generate logical contradiction within the host language, then I think the answer is a resounding no for anything that deserves to be called the liar sentence, and that to me is the end of the liar paradox story.

The first step in pursuing this inquiry must accordingly be to ask again exactly what the liar sentence is, what it is trying to say, and how it can best be understood as trying to say it, beginning with its logical structure. I have argued that the liar sentence is any sentence that satisfies the logical requirements of (L). The liar sentence is supposed to say *of itself* that it is false, and, according to my definition of the liar sentence, a sentence meeting the logical requirements spelled out in (L), it is L that is said to be false in the consequent of (L)'s conditional. The liar says, in effect, *I* am false, so whatever is said to be false in the liar paradox context must be the liar sentence, which in (L) is sentence L. Sentence L says of itself that it is false by virtue of satisfying the definition of the conditional liar sentence in (L).

If we suppose with Barker that the liar sentence is L rather than (L), and that we must look into the deeper workings of L to understand what drives the sentence into logical paradox, then I think we are in for another wave of triviality problems. Suppose that L all by itself is a logically inconsistent construction, that can be unpacked or instantiated in a variety of ways as Barker proposes in the second main component of his paper. Then L is itself a disguised logical inconsistency, a contradiction. If so, then Barker is

[2] Such enhancements of languages like Peano arithmetic are generally hostile enrichments, undertaken for purposes other than that for which the language was originally developed. Peano's arithmetic is a good example, in which a language dedicated to representing the axioms of a reductive functional number series is enlarged until it is capable of formulating a diagonal sentence that says of itself that it lacks some important logical or semantic property. Concluding triumphantly then that the language all along suffers an expressive limitation seems a rather pyrrhic victory and a bit of unfair treatment of the helpless target language.

certainly but disappointingly right that the liar sentence can have any of an unlimitedly vast number of hidden internal logical forms, including the jaw-dropping $p \wedge \neg p$. Barker works enormously hard in the second part of his (2009) discussion to devise ingenious logical contradictions, any of which he says can be understood as a liar sentence. If, however, the central point of his interpretation of my (L) or his (2) is right, then he could have saved himself the trouble and simply posited $L \leftrightarrow [p \wedge \neg p]$, which with the same plausibility he could then claim says of itself that it is false.

If the latter equivalence is true, no matter what explicit syntactical form sentence L as an outright logical contradiction is asked to wear, then both the conditional liar in (L) and the biconditional liar in (L*), as I shall defiantly continue to call them, will again be trivially true. For then the liar sentence itself will immediately express a contradiction, and hence (classically) cannot possibly be true. If, however, the liar sentence cannot possibly be true, then there can be no deductively valid path from the liar sentence's falsehood to its truth, thus squashing the second dilemma horn that is supposed to produce the liar paradox from the liar sentence. If, indeed, $L \leftrightarrow [p \wedge \neg p]$, then it is an empty tautology to say that FALSE$\ulcorner L \urcorner$, and under such an analysis we cannot go back from FALSE$\ulcorner L \urcorner$ to TRUE$\ulcorner L \urcorner$, which Saul A. Kripke aptly describes as the liar sentence's semantic *instability* (Kripke, 1975).

Instead, we can then validly infer only both conditional sentences, $L \rightarrow$ FALSE$\ulcorner L \urcorner$ and $\neg L \rightarrow$ FALSE$\ulcorner L \urcorner$, which is to say simply yet again that L is unqualifiedly false. That is an acceptable conclusion to me because it expresses my original proposal that the liar paradox is resolved by recognizing that the liar sentence is simply and unqualifiedly false. It is what I mean by *denying* the liar sentence as a dissolution of the liar paradox. Such a conclusion can nevertheless not sit well with Barker. In the biconditional form of the liar, my (L*) or Barker's (3), the one that Barker says he favors, at least as a statement identifying the liar sentence if not the liar sentence itself, L^* would have to be true in order for $L^* \leftrightarrow$ FALSE$\ulcorner L^* \urcorner$ to be true, where to suppose that $L^* \leftrightarrow$ FALSE$\ulcorner L^* \urcorner$ is false would be a complete nonstarter in any attempt to derive an interesting paradox from L^*. That's fine for one of the paradox dilemma horns, namely, the one going conditionally from the assumption that the liar sentence is true, to the conclusion that then the liar sentence is false. The problem, as I emphasized originally in my (2007) paper, is trying to go conditionally in the opposite direction, in the second paradox dilemma horn, from the proposition that the liar sentence is false to the conclusion that therefore the liar sentence is true. What we have on the present Barker-inspired assumption is that the liar sentence is logically equivalent to an explicit contradiction. If that were true, then anyone who takes the liar paradox more seriously than I believe it deserves to be taken should immediately reject Barker's analysis, because it means that to consider the liar sentence as a source of paradox involves no more than it would to simply posit as a wff of the logic any explicit contradiction.

Here, minimally, is one main if not the principal source of my arguments and Barker's objections going past one another. Since Barker is criticizing my arguments, however, it's surely his obligation to direct his assault against the view I actually accept, and not against another one that he says he takes for granted on the assumption that 'everyone thinks that way.' I can unequivocally report that the interpretation he offers of the relation of definition (L) and sentence variable L is not at all what I have ever intended. I hope that in the course of this clarification I have also adequately explained my reasons as to why I would not accept his application of the distinction between the liar sentence and a sentence that says of the liar sentence that it is a liar sentence.

That Barker systematically applies this new faulty interpretation is clear when he later makes a comparable but understandable misattribution, arguing:

> Now I would dismiss the terms "conditional liar" and "biconditional liar" as merely infelicitous choices of words if Jacquette did not make remarks such as the following: "I maintain that the liar sentence is a conditional that says of itself that it is false: (L) $L \rightarrow \text{FALSE}\ulcorner L \urcorner$" (Jacquette, 2008, p. 144). Here Jacquette is plainly referring to the conditional $L \rightarrow \text{FALSE}\ulcorner L \urcorner$, and not the sentence L itself, as a Liar sentence. In other words, he appears to be claiming that (2), and not the sentence L that (2) contains, is a Liar sentence. (Barker, 2009, pp. 7-8)

The distinction may at first seem understandable, but in fact Barker's latest exposition merely reflects the same conflation that he makes all along in interpreting my definition (L). Sentence L by itself is not the liar sentence, considered in and of itself merely as a sentence variable. The liar sentence in my treatment is rather the self-referential construction, $L \rightarrow \text{FALSE}\ulcorner L \urcorner$, as definition (L) specifies, making (L) in its entirety the liar sentence. Otherwise, we shall have to suppose as Barker does that L has a hidden internal logical structure whereby somehow behind the scenes it manages to say of itself that it is false.

In contrast, on the present proposal, sentence L is an instance of the liar sentence only insofar as it satisfies the conditions for the liar sentence spelled out in definition (L), by virtue of saying of itself that it is false. This makes (L) a statement of the logical structure of the liar sentence, and hence in the relevant sense makes the liar sentence defined by (L) itself a conditional. Similarly, if I wanted to define a type of sentence that has the property of being true conditionally on some other property, and hence not asserting its own truth, then I would do so by means of a definition such as: (T) $C \rightarrow \text{TRUE}\ulcorner T \urcorner$. Here it is clear that any sentence T by virtue of satisfying the definition in (T) is supposed to be true, conditionally on the truth of C, whatever that might be. Nevertheless, it remains the case that the sentence said to be true in this non-self-referential truth-teller counterpart to the liar sentence is not the conditional T in its entirety, but only the sentence variable T. It is not all of (T), the conditional $C \rightarrow \mathrm{TRUE}\ulcorner T \urcorner$ that is said to be true by (T). The truth of T, however, according to definition (T), is conditional on the truth of antecedent C, and in that familiar sense the truth of sentence T is conditional, rather than unconditional, biconditionally dependent on some other proposition, or standing in yet another unspecified logical relation to yet another proposition. This is the same sense in which the liar sentence in my analysis itself can correctly be described as conditional. It is not the sentences in and of themselves presented as sentence variables that say of themselves that they are true or are false, nor does the conditional in (L) say that the conditional in (L) is false, but rather that the consequent of the conditionals in (L) and (T) are true only insofar as they satisfy the conditions for sentences of the type laid down in the conditionally formulated definitions (L) and (T).

Just as Barker reveals, he took his interpretation so much for granted that he did not bother until (2009) to make it explicit, so I never dreamed that Barker had been proceeding on the basis of his particular mistaken interpretation of my use of notation in my (2007) and (2008). I couldn't have anticipated before now that anyone would think I was propounding an account of the liar at a meta-linguistic level, merely saying something *about* the liar, rather than defining and exhibiting the logical structure of the liar sentence itself. I have no interest in accepting an analysis of the liar sentence that floats at the superficial level of meta-liar talk, but I expect my proposal to stand or fall as a definition and explication of the liar sentence.

Barker, in my view, is wrong to assert the variable logical structure of the liar sentence, as a conditional, conjunction, or so on. The same logical relation can be expressed in a formal system of logic with the traditional five noble truth functions in any of a variety of logically equivalent ways. Beyond that trivial sense, I would certainly not agree with Barker if he means to say that the logical structure of the liar sentence admits of multiple logically nonequivalent hidden internal structures, and that it could be any of these, provided only that what he apparently reads as the metatheoretical conditions described in (L) are satisfied. As a result, I see nothing wrong at all in the quotations from (Jacquette, 2008) with which Barker concludes this section: 'Later, in a discussion of Tarski, [Jacquette] writes: "The liar *paradox* in Tarski's (β) is biconditional, just as we would expect it to be, but the liar *sentence*, in Tarski's (α), and as I would also insist, is conditional rather than biconditional" (Jacquette, 2008, p. 148).' This is, indeed, precisely what I need and want to say.

At the end of his §2, Barker candidly observes:

> I have discussed this matter at such length because everything that follows depends on it. The arguments to follow will simply be incomprehensible if one fails to distinguish Liar sentences on the one hand from the sentences (2) and (3) on the other. (Barker, 2009, p. 8)

Barker is right to remark that all of his remaining criticism depends on this matter of interpretation. What he does not fully appreciate is the fact that we speak of the conditional truth of sentences as the truth of those sentences, provided they satisfy the conditions, and not of the logical structure of the definitions by which such conditions are imposed. Thus, there is no tension whatsoever in jointly asserting that sentence *L*, which is not itself logically conditional in structure, is not false *simpliciter*, but only conditionally false, as prescribed by definition (L).

4. Intensionality of Literal Self-Reference

Building on the faulty foundation of what he takes to be the distinction between my definition (L) in which reference is made to sentence *L*, Barker proceeds undaunted to draw a series of equally mistaken conclusions about the adequacy of my interpretation of the liar paradox and the solution I propose.

Having recognized that sentence *L* is the liar sentence but not allowing that *L* is only conditionally false by virtue of satisfying the requirements of definition (L), and noticing that sentence variable *L* in and of itself is syntactically atomic, lacking the internal structure needed to say anything whatsoever about itself, Barker now advances to the second major part of his criticism, in which he attempts to represent the hidden internal logical structure of *L* through which *L* says of itself that *L* is false. This, however, is a matter for which provision is already made in my definition (L), and to which, as far as I can see, Barker does not add anything essential in his efforts to lift the curtain on liar sentence *L*'s deeper logic.

What Barker says is formally interesting and instructive, and deserves to be considered on its own merits, regardless of its lack of foundation in an accurate interpretation and correct understanding of my use of liar sentence definition (L) in relation to liar sentence *L*. I personally find Barker's suggestions here intriguing because I am interested in fine-grained interpretations of first-person intentions expressed in third-person public communication. I will try to say briefly what parts of Barker's analysis I agree with and give reasons for not immediately getting on board with others of more crucial importance in Barker's account of a deeper or more general formulation of the liar sentence than I offer in my (L) or consider in (L*).

Barker offers the following sequence of definitions and inferences:

(8) $\neg\mathrm{TRUE}(a)$
(9) $a = \ulcorner\neg\mathrm{TRUE}(a)\urcorner$
(10) $\mathrm{TRUE}(\ulcorner\neg\mathrm{TRUE}(a)\urcorner) \leftrightarrow \neg\mathrm{TRUE}(a)$
(11) $\mathrm{TRUE}(a) \leftrightarrow \neg\mathrm{TRUE}(a)$

This argument has a certain force, and Barker with good reasons of his own considers it to be decisive. He introduces (9) as the fact that *a* denotes the sentence (8) $\neg\mathrm{TRUE}(a)$, whereby *a* says of itself, of sentence *a*, that it is not true. He adds: '(10) is simply the result of substituting the sentence $\neg\mathrm{TRUE}(a)$ for the schematic letter *A* in (4) [the Tarskian disquotational truth schema]' (Barker, 2009, p. 10). In the absence of (9), sentence (8) is not saying anything at all. It is not even a sentence, since we do not know until we get to (9) what *a* refers to in (8). The letter *a* functions entirely as a dummy sentence variable waiting for (9) to provide the content needed for *a* to graduate from its original status as a fill-in-the-blank sentence *variable* to the *name* of a specific sentence. This is already a serious equivocation in Barker's development of the argument, but one that we can let pass for the sake of philosophically more interesting problems.

Barker's questionable move is from step (10) to (11). What he says about the inference is: 'Now from (10) and (9) we may derive the following [(11)] using the substitutivity property of identity' (Barker, 2009, p. 10). However, Barker's (11) is *not* a *uniform* substitution of sentence $\ulcorner a\urcorner$ for $\ulcorner\neg\mathrm{TRUE}(a)\urcorner$ in sentence (10), as licensed by the identity in (9). The *uniform* substitution of these terms in (10) is instead the obviously unparadoxical restatement of the redundancy formulation of the Tarskian truth schema:

(11.1) $\mathrm{TRUE}(a) \leftrightarrow a$

Still more importantly, if $a \neq \ulcorner a\urcorner$, observing the standard use-mention distinction, despite the fact that *a* after (9) is identified as a sentence, then Barker's selective substitution of $\ulcorner a\urcorner$ for $\ulcorner\neg\mathrm{TRUE}(a)\urcorner$ violates the extensionality requirement whereby we respect the fact that a sentence is not identical with its name, expressed again as $a \neq \ulcorner a\urcorner$. Barker thus unknowingly runs into the dilemma whereby we either respect or ignore the distinction $a \neq \ulcorner a\urcorner$. If we respect the distinction, then Barker's wanton substitution syntactically does not go through. If we ignore the distinction, then we intersubstitute *within* the *intensional* context $\ulcorner a\urcorner$, which in general constitutes a deductively invalid inference. Compare: Mark Twain = Samuel Clemens (TRUE); The name ⌜Mark Twain⌝ contains 9 letters (TRUE); therefore (FALSE), The name ⌜Samuel Clemens⌝ contains 9 letters. Remarkably, Barker explains the substitution in his derivation of (11) from (10) in these terms, failing to recognize the deductive invalidity of any such inference: 'We have simply replaced the quotation name $\ulcorner\neg\text{TRUE}(a)\urcorner$ in (10) by the constant a, a move that is justified by (9)' (Barker, 2009, p. 11).[3]

The problem clearly is with Barker's assumption (9). Sentence a according to (9) is identical to the *name* of a sentence, of the sentence in particular that $\neg\text{TRUE}(a)$. The difficulty is apparent when we reflect that the *names* of sentences, unlike sentences themselves, but like the names of anything else, Tom or Mary, are neither true nor false. This means that, given (9), Barker's (10) is no longer a true or false sentence. The right wing of the biconditional in (10), $\neg\text{TRUE}(a)$, is itself either (i) neither true nor false, or (ii) true in the sense that a as the name of sentence $\neg\text{TRUE}(a)$, according to (9), is not true. As such, it is then equally true that

[3] Barker does not use the Quinean corner quote convention as I do. For me, to write $\ulcorner\ldots\urcorner$ is to declare an intensional off-limits zone for intersubstitution of coreferential terms and logically equivalent sentences containing the corner quote bracketed text. Barker in (2009) but also in (2008) does not observe the substitution-exclusion of these intensional contexts, but relies on freely substituting identicals in such contexts for all of his principal arguments in defense of what he presents as a new method of deriving the liar beyond conditional and biconditional formulations of the liar sentence. Like all other logicians I know, I regard all such substitutions into intensional contexts as strictly forbidden, and I criticize the deductive validity of Barker's arguments that make use of such unlicensed substitution. Barker explains his own understanding of the corner quotes in his metatheory excursus in these terms: '[T]he difference between the corner quote convention that I am adopting in this section, and the convention adopted by Jacquette (and by me elsewhere in this paper), is actually quite minimal. The only difference is that under the present convention, $\ulcorner F\urcorner$ denotes the Gödel number of the formula F instead of denoting F itself' (Barker, 2009, p. 16). Such an interpretation is far removed from my intention in representing intensionally specific contexts falling outside the domain of *salva veritate* substitutions. What is true, although Barker does not observe or acknowledge the fact, is that the corner quotation is appropriately used for Gödel numbering functions, in the sense that the Gödel numbering of the syntax of logical expressions is inherently intensional. The Gödel number of sentence $p \neq$ the Gödel number of sentence q if, syntactically, $p \neq q$, even when it is true that $p \leftrightarrow q$. If we Gödel-code p as 1, $\rightarrow$ as 2, $\vee$ as 3, and $\neg$ as 4, then, although $[p \rightarrow p] \leftrightarrow [p \vee \neg p]$, $g\ulcorner p \rightarrow p\urcorner = 2^1 \times 3^2 \times 5^1 = 90$, in Gödel's numbering method on the glossary provided, whereas $g\ulcorner p \vee \neg p\urcorner = 2^1 \times 3^3 \times 5^4 \times 7^1 = 236{,}250$. From $g\ulcorner p\urcorner$ or Barker's $\ulcorner p\urcorner$ we cannot deductively validly infer, respectively, $g\ulcorner q\urcorner$ or $\ulcorner q\urcorner$, even if $p \leftrightarrow q$, as long as, syntactically, $p \neq q$. This is to say that Gödel numbering expressions like those Barker uses are substitution-inviolable intensional contexts. I explain the intensionality of Gödel contexts in (Jacquette, 1987; 2002).

$\neg$FALSE(a), which is evidently *not* a paradox or an alternative formulation of the liar paradox. For there is no contradiction in the permitted inference:

(11.2) $\neg\text{TRUE}(\ulcorner\text{Mark Twain}\urcorner) \wedge \neg\text{FALSE}(\ulcorner\text{Mark Twain}\urcorner)$

Or, for that matter:

(11.2.1) $\neg\text{TRUE}(\text{Mark Twain}) \wedge \neg\text{FALSE}(\text{Mark Twain})$

The reason is that by all the conventional distinctions it simply does not follow from $\neg\text{FALSE}(\ulcorner\text{Mark Twain}\urcorner)$ that therefore $\text{TRUE}(\ulcorner\text{Mark Twain}\urcorner)$. Rather, a very elementary category mistake is perpetrated in Barker's (8)-(11). What is true instead is only that $\neg\text{TRUE}[(\ulcorner\text{Mark Twain}\urcorner) \vee \neg(\ulcorner\text{Mark Twain}\urcorner)]$, because $\neg(\ulcorner\text{Mark Twain}\urcorner)$ is not a wff. The same is true for Barker's a identified by (9) as the *name* of a sentence, rather than a sentence itself. Where the sentence itself is involved, no paradox is forthcoming; and where the sentence's name is involved there is no legitimate deductively valid *salva veritate* substitution such as Barker needs to go from (9) and (10) to (11). I conclude that, contrary to his assertion, Barker does not develop a third way, beyond the conditional and biconditional formulation of the liar sentence, writing: 'A Liar sentence is a sentence that says of itself that it is not true. Now the conditional (2) and the biconditional (3) are two ways to attempt to capture this idea formally, but there is also a third way, which Jacquette does not consider' (Barker, 2009, p. 10). The above objection should make it clear why I cannot seriously entertain the 'third way' Barker introduces of proving the liar that involves neither the equivalent of a conditional or biconditional liar sentence.

Barker's (9) is evidently the main source of invalidity. If we were logically permitted to perform substitutions in Barker's swashbuckling way, then from Barker's true assumption (9) we could directly infer the false proposition purporting to identify what are manifestly *not* identical names of any sentence:

(9.1) $\ulcorner\neg\text{TRUE}(a)\urcorner = \ulcorner\neg\text{TRUE}\ulcorner\neg\text{TRUE}(a)\urcorner\urcorner$

Barker somewhat backs off from the inference in (9)-(11), and acknowledges a difficulty in (9), when he writes: 'Of course, names per se do raise a number of philosophical questions in their own right, so perhaps there is some legitimate uncertainty about whether we are really entitled to assert (9). That turns out not to matter, however, since there is yet another version of the Liar that we need to consider. Logically, much of the work of names can be performed by definite descriptions' (Barker, 2009, p. 11). If the proposed criticism is correct, then the difficulty in Barker's efforts to resurrect the liar paradox in light of my objections is not only with sentence

(9) and its use of names for sentences, or 'strings of symbols,' but with Barker's careless use of substitution principles in deriving (11) from (10). Barker himself ironically identifies precisely the problem in his attempt at proving the liar, when he explains:

> Thus, the fact that (9) conflicts with the disquotational schema (4) constitutes something of a puzzle in its own right. An adequate treatment of the Liar, in its present manifestation, would have to explain why, contrary to all appearances, we may not use the name a to denote the sentence '$\neg$TRUE(a)': either that, or explain why we may deny at least some instances of (4), which is logically the only alternative to denying (9). (Barker, 2009, p. 11)

We have met precisely this explanatory burden in our diagnosis of the deductive invalidity in Barker's (8)-(11), in which the choice is not only to deny either (4) or (9), but also to scrutinize more carefully the illegitimate use of substitution in the move from (10) to (11) on the basis of (9).

What, then, of Barker's shift away from names of sentences to definite descriptions? This is also initially suspicious and ultimately unacceptable. Barker builds throughout this part of his argument on the groundwork he labors to establish in the essay's opening passages, as previously noted, in which he seeks to show that I have confused the liar sentence itself with other sentences that purport metaparadoxically to describe some of the liar sentence's logical and semantic properties. For the reasons detailed above I reject Barker's application of this distinction to my critique of the liar sentence and liar paradox, and so I do not consider myself obliged to refute what he offers in this extensive section of his commentary. Since I find it intrinsically interesting, however, and since it repeats in more complicated form many of the same mistakes I find throughout Barker's analysis, I shall consider some of the most obvious difficulties his revision of the liar paradox in definite description terminology entails.

Barker begins with a formula $D(x)$, 'whose only free variable is x' (Barker 2009, 11), and invites us to consider the sentence:

(9) $\exists x(D(x)\ \&\ \neg\mathrm{TRUE}(x))$

He continues:

> Now suppose further that $D(x)$ happens to be satisfied by exactly one object. Then the sentence (12) says, more or less, that this unique object satisfying $D(x)$ is not true. (I say "more or less" because (12) does not explicitly say that $D(x)$ is satisfied by a unique object; this will turn out not to matter for our purposes, however.) Suppose further that this unique object happens to be the sentence (12) itself. Then (12) is yet another sentence that refers to itself and says, of itself, that it is not true. The fact that $D(x)$ is satisfied uniquely by (12) may be expressed formally as follows:

(9) $\forall x(D(x) \leftrightarrow x = \ulcorner \exists x(D(x) \ \& \ \neg\text{TRUE}(x))\urcorner)$ (Barker, 2009, p. 12).[4]

Barker goes on to explain how (13) supports a contradiction like his previous 'third way' of producing the liar paradox. This time the trick is performed without the naming of sentences, but engineered instead by means of their definite description when uniquely satisfied by a particular liar sentence. The problem, however, was never really with names as such anyway, but rather with Barker's cavalier use of identity substitution principles. The same objections I previously raised to Barker's 'third way' of deducing the liar paradox accordingly continue to hold sway here.

Notably, in (13) Barker makes an unwarranted substitution of sentence L for $\exists x(D(x) \ \& \ \neg\text{TRUE}(x))$ *within* the *intensional* naming or mentioning context, $\ulcorner \exists x(D(x) \ \& \ \neg\text{TRUE}(x))\urcorner$, to produce:

(13') $\forall x(D(x) \leftrightarrow x = \ulcorner L \urcorner)$

This maneuver is only permitted if the $\ulcorner \ldots \urcorner$ context is treated as referentially transparent, which for all the familiar reasons is prohibited by Barker's formulation of (13). This is clear when we consider, even as a purely syntactical matter, that, unmistakably, $\ulcorner L \urcorner \neq \ulcorner \exists x(D(x) \ \& \ \neg\text{TRUE}(x))\urcorner$, even if and whether or not it is also true that, equivalently, $L \leftrightarrow \exists x(D(x) \ \& \ \neg\text{TRUE}(x))$. Nor can Barker bite the bullet here. If the substitution context on which Barker relies were referentially transparent, then (13) would obviously contain a fatal quantifier collision, in which both the universal and existential quantifiers simultaneously bind all occurrences of the same variable 'x' in the subformula, $\exists x(D(x) \ \& \ \neg\text{TRUE}(x))$. Quantifier collision is contrary to standard predicate-quantificational formation rules, depriving Barker's (13) of any status as a wff, whether or not we 'abbreviate sentence (12) as L'. Nor will it avail to rewrite the subformula by uniformly replacing 'x' with 'y'. If we do that then we lose the self-referential

[4] Note that we can only regard Barker's (13) as well-formed, with no collision of the universal and existential quantifiers binding variable x in the subformula $\ulcorner \exists x(D(x) \ \& \ \neg\text{TRUE}(x))\urcorner$, if, in $\forall x(D(x) \leftrightarrow x = \ulcorner \exists x(D(x) \ \& \ \neg\text{TRUE}(x))\urcorner)$, we take the subformula to be nothing but the intensionally specific designation of a particular symbol string enclosed in corner quotes. We preserve (13) as a wff of predicate-quantificational logic only if we consider the subformula enclosed in corner quotes to occur within a substitution-inviolable intensional context. Barker in one place notes that the symbols enclosed in the corner quotes are a *specific* symbol string, yet, as argued in the body of the text above, he offers arguments in which he nevertheless freely intersubstitutes identicals within precisely these corner quote intensional contexts. Barker's deductively invalid *salva non veritate* substitution practice is the basis in turn for my rejection of his effort to advance a novel defense or 'third way' of proving the liar paradox without appeal to either an explicitly conditional or biconditional liar sentence.

ingredient that is indispensable to the liar sentence and liar paradox. Once again, Barker fails to respect the substitution blockade within intensional contexts in order to engage in fast and loose substitutions of identicals, giving the false appearance of having derived the liar paradox in a new way.

What is supposed to be novel in this part of Barker's discussion, in which he substitutes for *naming* the unique *description* of one form of the liar sentence, is that it gets us around objections to the use of names in deriving the liar paradox from a liar sentence that neither conditionally nor biconditionally denies its own truth. The trouble, as we have diagnosed it, is nevertheless much more deeply seated than the superficial syntactical choice of names or definite descriptions to designate the liar sentence. With Barker we may now elect also to designate the liar sentence by means of a description rather than a name. What does this accomplish? The alteration does not overcome the main objection that in order to produce a liar paradox conclusion of the form in Barker's (14) $L \leftrightarrow \neg\text{TRUE}(\ulcorner L \urcorner)$, Barker must make a deductively invalid substitution into an intensional context. I argued in both my papers to which Barker refers that from anything that deserves to be called the liar sentence we cannot logically deduce the equivalent of Barker's (14), but only at best the conditional going in the direction $L \rightarrow \neg\text{TRUE}\ulcorner L \urcorner$. We are therefore in a sense at the most important point of Barker's critique of my solution to the liar paradox. If he can prove (14), then he disproves my way of avoiding the liar, fair and square.

We should start to smell fish when Barker substitutes L for $\exists x(D(x)$ & $\neg\text{TRUE}(x))$ in presenting (13') as a more 'convenient' expression of (13). The rest of Barker's argument proceeds thereafter in terms of L, the *proper name* of an intensionally specific sentence, rather than in terms of the definite description that was supposed to rescue us from the potential but unidentified objections to using names in deducing the liar sentence purely by substitution of identicals in permitted, supposedly purely extensional, truth-preserving contexts. The only obvious defense of such a strategy must be to the effect that if the proper name for a liar sentence can be introduced by means of definite description, then the proper name formulation has all the virtues of the definite description version, to which it can always be reduced. This is true enough in purely extensional contexts, but not reliably where intensional contexts are concerned. Intensional contexts of the relevant kind are indeed defined precisely by their failure universally to sustain intersubstitution of coreferential terms or logically equivalent sentences *salva veritate*. If we grant Barker (13) or (13'), with the above understanding, how does he propose to deduce his decisive conclusion (14)? He argues:

(13') in turn logically implies

(9) $L \leftrightarrow \neg\text{TRUE}(\ulcorner L \urcorner)$

> To see that (13) implies (14), let us first rewrite (14) by expanding the left hand side of the biconditional:
>
> (14') $\exists x(D(x) \,\&\, \neg\text{TRUE}(x)) \leftrightarrow \neg\text{TRUE}(\ulcorner L \urcorner))$
>
> To show that (13) implies (14), we need only show that (13') implies (14'). Now (13') simply says that the sentence L is the unique object satisfying $D(x)$. Thus, both the left-to-right and the right-to-left direction of (14') follow directly from (13'). (Left-to-right: if $\exists x(D(x) \,\&\, \neg\text{TRUE}(x))$, then since L is the unique object satisfying $D(x)$, it follows that $\neg\text{TRUE}(\ulcorner L \urcorner)$. Right-to-left: if $\neg\text{TRUE}(\ulcorner L \urcorner)$, then since L satisfies $D(x)$, it certainly follows that $\exists x(D(x) \,\&\, \neg\text{TRUE}(x))$.) Thus, (13) logically implies (14). (Barker, 2009, p. 12)

Why such a complicated proof for what ought to follow, if it follows at all, in a logically straightforward fashion from explicitly defined concepts? Barker seems to lose command of his own definitions and their implications precisely here. What *is* the description $D(x)$ in (13) a description *of*, to *what* object or objects does it apply? It is the description of a particular intensionally specified *sentence*, $\ulcorner \exists x(D(x) \,\&\, \neg\text{TRUE}(x)) \urcorner$, which is the only thing to satisfy the description. When Barker proceeds to argue in the left-to-right attempt at justification he provides above for the inference of (13') to (14'), by beginning, 'since L is the unique object satisfying $D(x)$, it follows that $\neg\text{TRUE}(\ulcorner L \urcorner)$', he takes a disastrous first misstep. For L is *not* the unique object satisfying $D(x)$, by Barker's own definition of D in (13), but rather only and specifically, the liar sentence, as he now formulates it, $\ulcorner \exists x(D(x) \,\&\, \neg\text{TRUE}(x)) \urcorner$. At most, Barker could then logically derive:

$$\forall x(D(x) \leftrightarrow x = \ulcorner \exists x(D(x) \,\&\, \neg\text{TRUE}(x)) \urcorner) \rightarrow \neg\text{TRUE}(\ulcorner \exists x(D(x) \,\&\, \neg\text{TRUE}(x)) \urcorner)$$

This, however, is not what Barker's argument needs. It is precisely the liar paradox dilemma horn, from the truth of the liar sentence to its falsehood, that I acknowledge anyway in less complex terms, already in my (2007). Note that if Barker's (13) is true and we can validly detach the conditional's consequent, $\neg\text{TRUE}(\ulcorner \exists x(D(x) \,\&\, \neg\text{TRUE}(x)) \urcorner)$, then we are in effect denying the truth of the liar sentence, which is nothing other than my original proposal for avoiding the liar paradox by denying the liar,.

A similar mistake is made by Barker in the right-to-left justification of the biconditional. Here Barker argues: 'if $\neg\text{TRUE}(\ulcorner L \urcorner)$, then since L satisfies $D(x)$, it certainly follows that $\exists x(D(x) \,\&\, \neg\text{TRUE}(x))$'. We have seen that sentence L, even if it is logically equivalent to $\ulcorner \exists x(D(x) \,\&\, \neg\text{TRUE}(x)) \urcorner$, does *not* satisfy the precise terms of description $D(x)$ given in Barker's (13). The appropriate inference cannot soundly proceed from the proposition that $\neg\text{TRUE}(\ulcorner L \urcorner)$ satisfies $D(x)$, and as such cannot build the necessary logical bridge to $\exists x(D(x)\ \&\ \neg\mathrm{TRUE}(x))$, *unless* we understand Barker as violating the restrictions on deductively valid intersubstitution of terms or sentences in intensional contexts, such as we find spelled out in Barker's (13), as the specific symbol string $\ulcorner\exists x(D(x)\ \&\ \neg\mathrm{TRUE}(x))\urcorner$.

Accordingly, Barker's attempt to prove the right-to-left half of the biconditional by which he proposes to demonstrate that (13) implies (14), fails. It fails, moreover, in precisely the same way as his first version involving the intensionally specific *naming* of a liar sentence, on grounds of deductively invalid application of the substitution rules for identicals, and in particular of the rules prohibiting such substitutions *salva veritate* within intensional contexts like that presented in Barker's (13). Barker does not anticipate this line of objection in his penultimate paragraph on the topic, when he writes:

> Now if we decide to reject (13), we face essentially the same problem we faced when we contemplated rejecting (9). Namely, if the formula $D(x)$ happens, as a matter of empirical fact or otherwise, to be uniquely satisfied by the string of symbols $\exists x(D(x)\ \&\ \neg\mathrm{TRUE}(x))$, then we are automatically committed to (13). To reject (13), we must somehow show that a formula $D(x)$ can never uniquely describe the sentence $\exists x(D(x)\ \&\ \neg\mathrm{TRUE}(x))$. Yet $D(x)$ could easily turn out to do so. (Barker, 2009, p. 13)

The issue is not whether or not to accept the truth of (13) (or (9)). It is rather a matter of what can and cannot be validly deduced from (13), or, as before, from (9). Barker recalls immediately above that the description $D(x)$ innocently describes a particular 'string of symbols.' In his arguments involving (13), however, he loses sight of this fact and considers description $D(x)$ as satisfying and being satisfied by things other than that particular symbol string. This is the flaw in the proofs Barker proposes for both directions of the biconditional he proposes to defend, by which (13) implies (14) iff (13') implies (14').

5. Misadventures in Classical Metatheory

I turn now to Barker's invocation of classical metatheory. As I argued in (2008), I cannot accept what Barker says about Gödel's 1931 first metatheorem. Here I shall look only at the new discussion that appears in Barker's (2009, section 5), although I shall repeat some of the same previously voiced objections.

Barker maintains:

> The considerations of the last section can actually be used to show that the disquotational schema is logically incompatible with Peano Arithmetic, subject to a certain convention about quotation names. I made this point all too briefly in (Barker, 2008), and the point was greeted with incredulity: "[Barker] actually claims to be able to deduce (L*) as a substitution instance for a theorem of Peano

> Arithmetic!" (Jacquette, 2008, p. 151). Jacquette's (L*) is simply my (3), and yes, I do claim to be able to derive (an instance of) (3) from Peano Arithmetic. That this can be done is in fact quite well known, although evidently not quite well known enough. The basic technique can be found in (Gödel 1931), though Gödel does not actually derive an instance of (3) there. (Barker, 2009, p. 15)

There are important historical and philosophical questions raised by Barker's assertions.[5] Barker strays beyond the limits of Peano arithmetic early in his argument when he adds a 'single unary predicate TRUE' to **L** as the 'language of arithmetic' to provide **L'** (Barker 2009, p. 15). There is no truth predicate in Peano arithmetic, but at most only in its metatheory, which is another thing altogether. Barker strangely acknowledges this, stating: 'This is all the more remarkable given that the predicate TRUE does not even belong to language **L** in which Peano Arithmetic is formulated' (Barker 2009, 15). Remarkable, indeed, and once again quite ironic. Since Barker recognizes that languages **L** and **L'** are distinct in precisely the respect that Peano arithmetic formulated as **L** does *not* contain the predicate TRUE that his argument requires, what he proposes is like diagnosing patient *A* from patient *B*'s x-rays and blood work. Barker, dauntless, nevertheless proceeds: 'Then there is indeed a sentence *L* of the language **L'** such that the biconditional $L \leftrightarrow \neg\mathrm{TRUE}(\ulcorner L \urcorner)$ is a theorem of Peano Arithmetic, Jacquette's skepticism in this matter notwithstanding' (Barker 2009, 15).

I freely admit that my skepticism in the matter has not abated, especially after reading Barker's most recent remarks. I still do not think, based on what he says in his (2008; 2009), that Barker correctly understands Gödel's 1931 results or Peano arithmetic. Barker then invokes the 'key to proving this remarkable result is the following, even more remarkable result, from (Gödel, 1931), which is variously known as the "diagonal lemma" and the

[5] I take the opportunity, in light of Barker's remarks about his expectations concerning my knowledge of standard metatheory, to defend myself against the image that Barker polemically encourages of my antagonism in responding to some of his previous criticisms. He writes, early in his second paper: 'Suffice it to say that Jacquette was not at all convinced by my arguments; indeed, he characterizes my paper as "riddled from beginning to end with confusions and misunderstandings" (Jacquette, 2008, p. 143)' (Barker, 2009, p. 5). I said this, of course, and I still think it is true. However, the full context of my remark, including an important part of the comment that Barker does not quote but omits, was actually: 'Barker twice in the [2008] essay accuses my reasoning of being "flawed," but Barker's own exposition is riddled…etc.' (Jacquette, 2008, p. 143). I was not trying to be unduly hostile, but believed that I was merely repaying Barker's compliment in an exchange Barker initiated and for which he had set the tone.

"self-reference lemma"' (Barker, 2009, p. 15). What follows next virtually takes my breath away, as Barker continues:

> Letting **L'** again be any countable extension of **L**, and letting $F(x)$ be any formula of the extended language **L'**, there is a sentence A, also of **L'**, such that the following is a theorem of Peano Arithmetic:
> (22) $A \leftrightarrow F(\ulcorner A \urcorner)$
> Finally, choosing the formula $\neg\mathrm{TRUE}(\ulcorner x \urcorner)$ for $F(x)$, we see that there is a sentence L of **L'** such that the biconditional $L \leftrightarrow \neg\mathrm{TRUE}(\ulcorner L \urcorner)$ is derivable from Peano's Arithmetic: this is simply a direct consequence of the diagonal lemma. (Barker, 2009, p. 16)

Barker explains:

> I used the diagonal lemma in just this way in (Barker, 2008). However, I did not bother to prove the diagonal lemma or provide a full citation, because frankly I thought it was something every logician knew. However, Jacquette *seems* to take issue with it, if I am correctly understanding the argument of section 10 of his paper. Be that as it may, the diagonal lemma is an entirely standard result, though different authors present the result in slightly different ways. The following quote from (Boolos & Jeffrey, 1980) is fairly typical:
> Here's the diagonal lemma:
>
> **Lemma 2**
>
> Let T be a theory in which diag is representable. Then for any formula $B(y)$ (of the language of T, containing just the variable y free), there is a sentence G such that $\vdash_T G \leftrightarrow B(\ulcorner G \urcorner)$ (Boolos & Jeffrey, 1980, p. 173). (Barker, 2009, p. 16)

If you start out with Peano arithmetic and supplement it with enough syntactical apparatus so as to be able to formulate a sentence like G, then, sure enough, there does indeed exist such a sentence G in the expanded language. The question is, how do we get from there to Barker's conclusion that his (22) above is 'a theorem of Peano arithmetic'? If the method were sound, then we could do the same with the set of all facts about the American Civil War, for example, thereby proving it a 'theorem' of history that all facts about the American Civil War are jointly logically inconsistent, as well as being jointly logically consistent, and, in fact, that they have any property and its complement whatsoever. The particular application that Barker wants to make of what he calls Gödel's diagonal lemma is definitely too rich for my palate.[6]

[6] Boolos and Jeffrey would not be my first choice as expositors of contemporary logical metatheory, but since Barker refers directly exclusively to them and only obliquely to Gödel and Tarski, I shall cast my replies in Boolos and Jeffrey's notation for the diagonal lemma in their (1980). Note that whereas in (Jacquette 2008) I had called upon Barker to support his assertion that *Gödel* had proved a substitution instance of (L*) (Barker's sentence (3)) with citations and quotations, Barker still has not done so, referring instead to Boolos and Jeffrey's

I do not read Gödel or Boolos and Jeffrey as making nearly so categorical and latitudinarian an assertion as Barker attributes to them. Boolos and Jeffrey, in the passage Barker cites, merely assert the existence of a certain diagonal sentence G forthcoming as a theorem for any choice of predicate B of language T. They generalize universally over any predicate B, but they do not say that the relation holds true of *any* sentence. Barker similarly begins by saying that there is a sentence A such that $A \leftrightarrow F(\ulcorner A \urcorner)$. He chooses $\neg\text{TRUE}(\ulcorner x \urcorner)$ for $F(x)$, which should take him only to the conclusion that $A \leftrightarrow \neg\text{TRUE}(\ulcorner A \urcorner)$. This is reasonable enough, provided that A like G is a diagonal sentence constructed accordingly to Boolos and Jeffrey's requirements. There are such diagonal sentences, but in standard metatheory we are not permitted to infer that just because G is constructible that therefore G is true or that we can replace another sentence like the liar sentence L for G.

We must remind ourselves that the sentence G or A in question is only *some* sentence for which the biconditional holds true, and in particular a diagonal sentence. When Barker makes the crucial substitution from $A \leftrightarrow \neg\text{TRUE}(\ulcorner A \urcorner)$ to $L \leftrightarrow \neg\text{TRUE}(\ulcorner L \urcorner)$, he goes beyond Boolos and Jeffrey's proof of the original sentence generalized for any property of diagonal sentence G. To proceed as though the liar sentence L were also diagonal is to assume that the liar sentence is such that $L \leftrightarrow \neg L$, which begs the question as to the proper analysis of the liar sentence's underlying logical form against the conditional interpretation of the liar that I have proposed. The question is not whether it is possible in an appropriately strengthened system of logic to construct a diagonalized sentence such as Boolos and Jeffrey's sentence G. This by itself is indeed a recognized part of classical metatheory since Gödel. We know from Gödel also that there is then a dilemma, according to which G is either true in T or not. Since G for an appropriate property B says of itself via Boolos and Jeffrey's lemma that it is not provable, if G is true then it is not provable, rendering logics like T powerful enough to express diagonal sentence G undecidable or deductively incomplete; whereas if G is false, then it is possible to prove false propositions in T, ultimately making T inconsistent. I do not deny that Barker's $A \leftrightarrow \neg\text{TRUE}(\ulcorner A \urcorner)$ is a *theorem* of an expanded Peano arithmetic,

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Lemma 2. Historical niceties aside, it is worth remarking that Barker does not try to ground his assertions in Gödel's (1931) essay. This is reasonable, because Gödel's text does not support Barker's reading and application. I find it interesting that in this context Barker now significantly demurs, qualifying his claim: 'The basic technique can be found in (Gödel, 1931), though Gödel does not actually derive an instance of (3) [(L*) in (Gödel, 1931)] there' (Barker, 2009, p. 15). I explain my concept of diagonalization, and its applications in the Gödel, Church and Rosser proofs, in (Jacquette, 2002).

but I repeat, contrary to Barker's unsupported claim, that, reverting to the notation in which I define L^* as the biconditonal liar sentence, that $L^* \leftrightarrow \neg\mathrm{TRUE}(\ulcorner L^* \urcorner)$ is *not*, in Barker's words, '*provable*' (Barker, 2008, p. 139).

This is the assertion to which I was referring in Barker's (2008), which has now occasioned Barker's (2009) lesson in classical metatheory. Barker says:

> So why not simply deny (L*)? Wouldn't that be better than denying (TS) [Tarskian truth schema]? It would be, except that we are stuck with (L*): it is *provable*. (Barker, 2008, p. 139).

I think the assertion that (L*) is *provable* goes too far, and this was the substance of my objection to Barker's original contention. Suppose that (L*) is defined as $L^* \leftrightarrow \neg\mathrm{TRUE}(\ulcorner L^* \urcorner)$, a point on which Barker and I agree. Now suppose as Barker alleges that (L*) is provable. If (L*) is provable, then the two component conditionals of (L*), (a) $L^* \rightarrow \neg\mathrm{TRUE}(\ulcorner L^* \urcorner)$ and (b) $\neg\mathrm{TRUE}(\ulcorner L^* \urcorner) \rightarrow L^*$, are provable. I have no quarrel with the provability of (a), which, indeed, I have defended on intuitive and formal grounds (as the *only* validly derivable component of the liar paradox from a properly formulated liar sentence, already in (Jacquette, 2007). The difficulty is proving (b), effectively the second horn of the liar paradox, as I have also emphasized from the outset in my analysis of the liar sentence and solution to the liar paradox.

Let us accordingly consider what is involved in proving (b). Since it is a conditional, we should expect there to exist a deductively valid proof in which the assumption is $\neg\mathrm{TRUE}(\ulcorner L^* \urcorner)$ and the conclusion is L^*. Alternatively, we can consider the conditional (b) to be provable iff the consequent or the negation of the antecedent is provable. The negation of the antecedent in this case is $\neg\neg\mathrm{TRUE}(\ulcorner L^* \urcorner)$, or simply, $\mathrm{TRUE}(\ulcorner L^* \urcorner)$. By the Tarskian truth schema this also amounts to L^*, so we concentrate on it in considering whether or not (L*) is provable. Next, we define the property of being *un*provable, as follows:

(DFUP) $\forall\varphi[\neg\mathrm{PROVABLE}(\varphi) \leftrightarrow \neg\, \exists x[x \vdash \varphi]]$

We include the null set $\varnothing$ as a possible substituend for the existentially bound variable x to cover the case of tautologies for the unnegated existential. Next, we borrow a page from Barker's playbook, assuming he gets his metatheory right, and assume that L^* is in some sense diagonal or can be Gödel-diagonalized, and instantiate the Boolos and Jeffrey diagonal lemma in this way:

(UP) $\vdash_T L^* \leftrightarrow \neg\mathrm{PROVABLE}(\ulcorner L^* \urcorner)$

If (UP) is also a *theorem* of the expanded Peano arithmetic *T*, subject to Boolos and Jeffrey's diagonal lemma, as Barker understands and unreservedly applies it, then any attempted proof of (L*) from any assumption whatsoever, including $\neg$TRUE($\ulcorner L^* \urcorner$), logically cannot succeed. This result effectively undermines the proof of (b) Barker requires in order for (L*) to be *provable*. To rephrase things somewhat more accurately, any attempted proof of (L*) in *T* from any assumption whatsoever, including $\neg$TRUE($\ulcorner L^* \urcorner$), logically cannot succeed, provided that *T* is logically consistent. Boolos and Jeffrey themselves assume throughout that the classical logic for which they develop a metatheory is itself logically consistent, just as it is for Peano arithmetic, to deny which would trivialize any and every conclusion of the metatheory concerning the logic's properties.[7]

We can work the same ploy for the property PROVABLE, *if* Barker correctly applies the implications of Boolos and Jeffrey's lemma. This is just the problem where diagonal sentences are concerned, and why my objections to the liar paradox can be construed as skepticism about whether the liar sentence is fully diagonal. For Peano arithmetic to be capable of *proving* (L*), as Barker asserts, would accordingly amount to declaring that the theory is unqualifiedly logically inconsistent. That outcome, however, is only one of two possibilities countenanced by the original Gödel undecidability result, whereupon the Gödel sentence despite diagonally asserting its own unprovability becomes classically trivially provable by paradox of material implication. If I understand Barker's reasoning, then he attempts incorrectly to force the Gödel dilemma in that single trivializing direction, blaming the whole mess on the Tarskian disquotational truth schema. That is nevertheless a radical choice that to my knowledge is not endorsed by Gödel, Boolos and Jeffrey, or any of the other responsible commentators on the classical limiting metatheorems. All competent logicians to my knowledge grasp the other horn of Gödel's dilemma, since there is thankfully a choice, and conclude instead that the diagonal sentences constructible in languages sufficiently fortified in order to be able to formulate them are true but unprovable, raising further interesting logical and philosophical questions, devising and opening the door to a variety of controversial philosophical applications, but at least preserving syntactical

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[7] Boolos and Jeffrey, to continue with Barker's source, develop their metaproof of the undecidability and deductive incompleteness of a language capable of expressing function diag by means of **Lemma 3**: 'If *T* is a consistent extension of *Q*, then the set of gödel [sic.] numbers of theorems of *T* is not definable in *T*' (1980, 174). See also their **Theorem 1** and **Lemma 4**, in the immediately ensuing pages.

consistency. This is why I previously wrote, and why I remain stunned at Barker's application:

> If Gödel's 'famous' lemma, as Barker characterizes it, entailed that all of Peano arithmetic, couched in classical logic and supplemented merely by a Tarskian truth schema, was out-and-out logically inconsistent, instead of being either syntactically inconsistent or deductively incomplete, then I think that Gödel's lemma and his proof would be even more famous than they are. (Jacquette, 2008, p. 154)

We are left in any case with the conclusion, contrary to the impression Barker cultivates, that (L*) is not simply *provable*, as *opposed* to being *un*provable, in a suitably fortified system of logic like Peano arithmetic. If his understanding of classical metatheory were correct, then its application undermines his own efforts to prove the liar paradox from his reformulation of the liar sentence. Nor is there any reason to single out Peano arithmetic as subject to such a formal indignity, since the same would then be true of any logical system or body of knowledge, such as the facts of the American Civil War, if only we are permitted to add enough kit to the original set of propositions so as to facilitate diagonalization. If I have misunderstood Barker on this central point, then I gladly take back all these objections — but then what Barker seems to have been asserting in maintaining that (L*) is unequivocally *provable* in Peano arithmetic, and that we are therefore 'stuck' with (L*) (Barker, 2008, p. 139), is drained of any of its initial apparent interest.

It is not because I am spectacularly uninformed about the mysteries of classical metatheoretical results that I balk so demonstrably against Barker's references to the diagonal lemma, but because his attempt to apply the lemma gratuitously saddles all of classical logic capable of formulating diagonal sentences through and through with outright logical inconsistencies. That drastic interpretation, as far as I know and for obvious reasons, is not seriously entertained by Gödel or by any of his later expositors. My astonishment at Barker's pronouncements specifically about the *provability* of (L*), as opposed to (L), is predicated on my objection that Barker takes things too far, and, without justification, overrides the standard reaction to the diagonalization dilemma in favor of an indefensible thesis concerning the *provability* of (L*) that would land all of Peano arithmetic in outright contradiction. He seems to think that everyone but me already knows this, but he doesn't exactly wallpaper his essay with substantiating endorsements from the logic community for his interpretation. I say again, therefore, that such a construal of (L*) as deductively provable in Peano arithmetic is logically unwarranted, even if (L*) or an equivalent arithmetized diagonalization were to be regarded as a proper formalization of the liar sentence, which, again, I emphatically deny.[8]

As a concluding historical note to this section, Peano's own formulation of arithmetic, as Barker surprisingly acknowledges, is *not* capable of representing Boolos and Jeffrey's diag function. Barker finds it 'remarkable' that, despite this fact, Peano's arithmetic itself is subject to the provability of (L*) interpreted as a diagonal sentence. I would rather say that any such attribution is unintelligible. The limits of **L'** are not necessarily the limits of **L**, even if we define **L'** by beginning with **L** and adding devices to it that eventually run it into conflict with the limits (no lectures about conservative extensions in Church's Theorem at this point, please) of **L'**. We might as well then try to attribute the undecidability or deductive incompleteness of classical first-order predicate-quantificational logic to propositional logic, since standard first-order predicate-quantificational logic is also an extension or expansion of a propositional base logic. I oppose Barker's assertion of the provability of (L*) in Peano arithmetic **L** for this reason also.

6. Understanding the Liar

Finally, I want to supplement my previous (2007; 2008) intuitive remarks about the liar sentence saying simply that 'I am false' can only be rightly understood as a conditional and not as a biconditional assertion. This is one of the items on what Barker calls my 'laundry-list' reply to his (2008) that Barker does not choose to address. An unfortunate omission, because I place an enormous amount of weight on precisely these considerations in

[8] The disquotational Tarski truth schema is not used in Gödel's original proof, and, given the chronology, could hardly have been known by Gödel in 1931 when he crafted the undecidability argument against *Principia Mathematica* and related systems. This casts further doubt on Barker's assertions (2008; 2009) that it is the Tarski truth schema in particular that is responsible for the liar paradox and for the mayhem he projects upon the diagonal-expressive modifications of Peano arithmetic. Barker gets it wrong again when he conjectures that I am somehow wedded to Tarski's truth schemata, writing: 'it is entirely understandable that Jacquette would prefer to deny just about any statement rather than deny the [Tarskian] disquotational schema' (2009, p. 20). This is untrue. I am in fact only too interested in rejecting Tarski's truth schemata on any legitimate formal or philosophical grounds. My philosophical dissatisfaction with Tarskian truth schemata for reasons independent of Barker's complaints can be found already in (Jacquette, 2002; 2009; 2010). I just do not agree that the particular conclusions Barker tries to support in his arguments can in this case be correctly attributed entirely or even largely to the responsibility of Tarski's truth schemata, and I do not regard Barker's argument as forcing their repudiation.

arguing that the liar sentence must be conditional rather than biconditional in form, by virtue of which it is formally too weak to uphold a logical contradiction.

The liar sentence in any of its accepted incarnations clearly does not say, biconditionally: 'I am false if and only if I am true', 'I am true if and only if I am not true', or 'I am false if and only if it is not the case that I am false'. Any such formulation of the liar sentence would obviously make the liar paradox built on the sentence altogether trivial, like simply asserting $p \wedge \neg p$. In the case of the aboriginal Epimenides paradox in the Gospel of St. Paul, Epistle to Titus 1:12, the proto-liar sentence says only conditionally, as spoken by Epimenides, a Cretan, (C): 'All Cretans are liars'. This is a universal sentence, and universals are generally formalized by means of a conditional rather than biconditional, here as $\forall x[\text{CRETAN}(x) \rightarrow \text{LIAR}(x)]$.

Cretans in the ancient world, incidentally, did not acquire the reputation for dishonesty through deceitful business practices or general lack of respect for the truth, but because many people on Crete believed at the time that the supposedly immortal god Zeus had died and was buried on the island. The mere profession of a *false* belief, however, assuming that the Cretans' belief was false, does not qualify as *lying*, nor does it imply that everything a Cretan might say thereafter should be considered a lie. In the Biblical formulation, the converse of the above conditional does not hold true, since no one supposes that *only* Cretans are liars, that $\forall x[\text{LIAR}(x) \rightarrow \text{CRETAN}(x)]$, or that *all and only* Cretans are liars, $\forall x[\text{LIAR}(x) \leftrightarrow \text{CRETAN}(x)]$. When we try to superimpose the liar argument on this informal version of the liar sentence, just as in the formal version criticized as falling short of a genuine contradiction, we do not collect a full paradox from both dilemma horns. If sentence (C) is true, as with (L), then it is false; whereas, if (C) is false, then (C) is only *possibly* true. From $\neg\forall x[\text{CRETAN}(x) \rightarrow \text{LIAR}(x)]$, it follows only that there is at least one Cretan who is not a liar, $\exists x[\text{CRETAN}(x) \wedge \neg\text{LIAR}(x)]$. The veracious Cretan whose existence is implied by the falsehood of (C) nevertheless might but need not be identical to Epimenides as the particular Cretan who utters (C), on which the self-reference of the original liar logically depends. It will clearly not do to argue that if (C) is false, then Epimenides, as a Cretan, by virtue of having lied, has spoken truly in asserting that all Cretans are liars, because logically the falsehood of (C) does not exclude the possibility that Epimenides does not lie on all occasions, let alone that all other Cretans always lie. He might or might not be telling the truth when he says falsely that all Cretans are liars. This is a limitation of the original liar sentence, which, consistently with the refutation of the liar that I have proposed, supports only half of the liar paradox in the first paradox dilemma horn.

If we switch to a biconditional reformulation of Epimenides, (C*), saying that 'All and only Cretans are liars', then, significantly, the situation logically is not improved. In that case, the falsehood of (C*), its not being the case that all and only Cretans are liars, implies that there exists at least one individual who is either a liar but not a Cretan, or a Cretan but not a liar, $\exists x[[\mathrm{LIAR}(x) \wedge \neg\mathrm{CRETAN}(x)] \vee [\mathrm{CRETAN}(x) \wedge \neg\mathrm{LIAR}(x)]]$. Even if we suppose that Epimenides was the only Cretan, which is neither factually true nor guaranteed by logic, it still would not follow logically from the falsehood of (C*) that Epimenides in particular was not a liar — only and at most that *some* Cretan was not a liar or that *some* liar was not a Cretan. As a consequence, it does not follow even for the amped-up version of Epimenides' statement, holding biconditionally that all and only Cretans are liars, that (C*) must turn out to be true on the assumption that it is false.

All this is to say, as I have maintained throughout (2007, 2008, and here), that the so-called liar paradox, formulated in terms of anything that deserves historically or philosophically to be called a logically interesting potentially paradoxical liar sentence, is not a genuine paradox in the sense of implying a logical antinomy or syntactical inconsistency. The liar sentence, properly interpreted, is conditional like (L), in which case it is not deductively powerful enough to imply the liar paradox, as Barker acknowledges. If the liar sentence is supposed to be biconditional like (L*) or diagonal, then the liar sentence only trivially implies the logical inconsistency of the liar paradox. This is the main item on my laundry list of complaints that I wish Barker had confronted. It is never surprising or formally or philosophically interesting in classical logic to pass deductively from one explicit contradiction to another, because such an inference is logically trivial. This consideration is surely enough to eliminate as interesting any candidate liar sentence that in and of itself already explicitly expresses a logical contradiction, thereby depriving the so-called liar paradox of any logical or philosophical interest.

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